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Dynamical symmetry breaking on a brane with bulk gauge theory

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Abstract

We analyze a structure of dynamical chiral symmetry breaking in orbifold gauge theories with matter fields (fermions) on the fixed point. We find that the boundary chiral phase structure of QED and QCD on the orbifold is quite nontrivial depending on the bulk constitution, and we claim that particular attention should be given to the dynamically generated masses in various kinds of phenomenological orbifold models.

It is revealed recently that the orbifold field theory can provide, e.g., the weak and Planck hierarchy [1], a fermion mass hierarchy (via localization [2]), a gauge symmetry and a supersymmetry (SUSY) breaking [3, 4] (by the Wilson line [5]), SUSY breaking mediation mechanisms (e.g., [6]), and the proton stability (e.g., [7]), based on perturbative (or tree) analyses. However an effective higher-dimensional theory such as the orbifold model means $M_c < \Lambda$ by definition, i.e., the compactification scale should be less than the cut-off scale of the theory. This implies an existence of Kaluza-Klein particles below Λ and we have a question about their effect on a nonperturbative dynamics of the theory.

One simple example for such a nonperturbative dynamics is a chiral symmetry breaking structure in a $SU(N)$ Yang-Milles and matter (fermion) theory on the orbifold. Assuming only a kink-type mass $\epsilon(y)M_{\rm kink}$, we have a chiral symmetry for the fermion zero mode. The fermion zero mode couples to KK gauge bosons as well as a gauge boson zero mode. So the chiral phase structure may be different from the usual 4D case (e.g., a four-fermion approximation, Ref. [8]). The simplest case for the analysis is $|M_{\rm kink}| \to \Lambda$ that means all the KK fermions decouple and the fermion becomes a brane field effectively. Also a lot of orbifold models introduces intrinsic brane fields (fermions) which couples to a bulk gauge boson. Therefore we analyze a dynamical chiral symmetry breaking (DSB) on a boundary with a bulk gauge theory.

We consider QCD (QED) on $M_4 \times S^1/Z_2$ with quarks (electrons) on the fixed point. For generality, the bulk geometry is assumed as, $ds^2 = G_{MN}dx^M dx^N = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$, where k is a AdS curvature scale. We use R, Λ and $\Lambda_{\rm 5D}$ as a radius of the fifth dimension, a 4D (brane) effective cut-off scale and a 5D cut-off scale respectively. The 4D effective Lagrangian is given by

$$\mathcal{L} \;=\; \frac{1}{2}\sum_{n=0}^{N_{\rm KK}} A_\mu^{(n)}\left[\eta^{\mu\nu}\left(\partial^2+M_n^2\right)-(1-\xi)\partial_\mu\partial_\nu\right]A_\nu^{(n)} + \bar\psi\left(i\partial_\mu + gA_\mu^{(0)}\right)\gamma^\mu\psi + g\sum_{n=1}^{N_{\rm KK}}\frac{\chi_n(y^*)}{\chi_0(y^*)}\bar\psi A_\mu^{(n)}\gamma^\mu\psi,$$

where $y^* = 0, \pi R$ represents orbifold fixed points. We neglected gluon self interactions which will be incorporated latter via a one-loop running coupling. The $\chi_n(y)$ and M_n is the n-th Kaluza-Klein (KK) mode function and the mass eigenvalue of the gauge field, respectively. The $\chi_n(y)$ takes a value of $\chi_n(y^*) = \sqrt{2}$ in a flat geometry, and less (more) than this value at $y^* = 0$ (πR) in a warped geometry, depending on the AdS curvature k. In the following analysis we take $kR = 11.35$ as a typical example. The ξ is a gauge fixing parameter which will be fixed in such a way that the chiral Ward identity is approximately held in the following numerical analysis.

To analyze DSB in our system, we solve a Schwinger-Dyson (SD) equation for a fermion propagator on the brane,

$$iS^{-1}(p) \;=\; iS_0^{-1}(p) + \sum_{n=0}^{N_{\rm KK}}\int\frac{d^4q}{i(2\pi)^4}\left[-ig_nT^a\Gamma^M\right]S(q)\left[-ig_nT^a\Gamma^N\right]D_{MN}^{(n)}(q-p),$$

where $S_0(p) = i/\not p$, $S(p)$ and $D_{MN}^{(n)}(k)$ stand for a free fermion propagator, a full fermion propagator and a full n-th KK gauge boson propagator, respectively. We parameterize the full fermion propagator as $iS^{-1}(p) \equiv A(-p^2)\not p - B(-p^2)$ and solve the SD equation numerically in terms of A and B.

For orbifold QED (Abelian) theory with $M_c \lesssim \Lambda$, we apply so-called ladder approximation to the SD equation which utilizes a tree vertex and a tree gauge boson propagator. The results from the ladder SD and a local four-fermion approximation are shown in Fig. 1. We conclude that the critical coupling $\alpha_c = g_c^2/4\pi^2$ is less than the usual 4D one, which means the KK modes enhance DSB, and also claim that the local four-fermion

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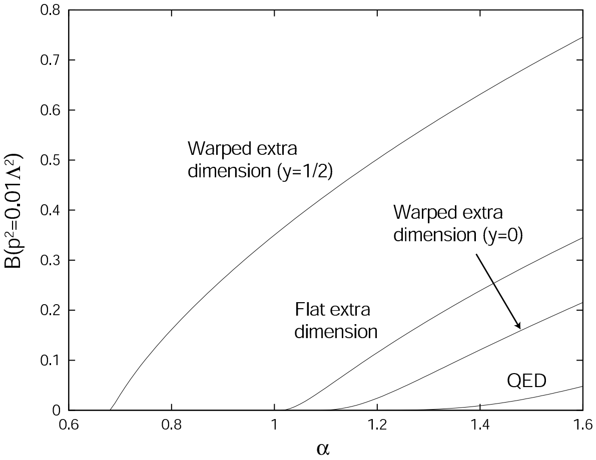
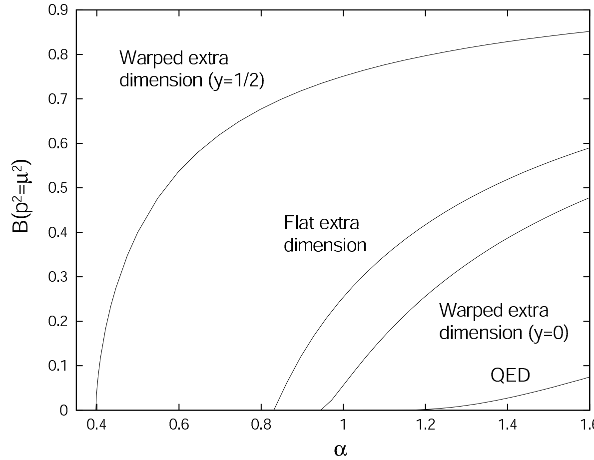


Ladder SD ($N_{\mathrm{KK}} = 1$)



Local four-fermion approximation ($N_{\mathrm{KK}} = 1$)

Figure 1: Chiral phase structure on a boundary of bulk QED.

approximation overestimates the KK effect on DSB, i.e., the KK propagation effect weakens DSB. The point (tree) vertex is sufficient? The $\alpha(\mu)$ runs indeed logarithmically, then it is negligible in QED (Abelian). And it runs in power low for $M_1 < \mu < \Lambda$, then it is also negligible for $M_1 \sim \Lambda$ ($N_{\mathrm{KK}} \simeq 1$). However, for QCD (Yang-Mills) and/or $M_1 \ll \Lambda$ ($N_{\mathrm{KK}} \gg 1$), we need to improve the ladder SD equation.

Then, next we develop so-called improved ladder approximation (e.g., Ref. [9]) in the orbifold QCD. The one-loop perturbative running coupling is given by a truncated KK analysis [10] as

$$\frac{\pi}{4}\alpha^{-1}(z) \quad = \quad B\ln(z/\Lambda_{\mathrm{QCD}}^2) - \widetilde{B}\left[\ln(z/\mu_R^2) - \frac{2X_\delta}{\delta}\left\{(z/\mu_R^2)^{\delta/2} - 1\right\}\right]\theta(z-\mu_R^2),$$

where $\mu_R = 1/R$, $B = \frac{1}{24C_2(F)}\left(\frac{11N_c - 2N_{\mathrm{f}}}{3}\right) = 9/16$ ($N_c = 3, N_{\mathrm{f}} = 3$), $\widetilde{B} = \frac{1}{24C_2(F)}\left(\frac{11N_c - 2\tilde{N}_{\mathrm{f}}}{3}\right) = 11/16$ ($N_c = 3, \tilde{N}_{\mathrm{f}} = 0$), $X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)}$ and δ is the number of extra dimensions. For a warped geometry we replace some quantities as $\Lambda = \Lambda_{5\mathrm{D}} \to e^{-ky^*}\Lambda_{5\mathrm{D}}$, $\mu_R \to \mu_{kR} = \pi e^{-\pi kR}k$, and $\sqrt{z} \to \sqrt{z} + \mu_{kR}/4$. For the orbifold QCD, the result of a fermion mass function $B(x)$ and the pion decay constant f_π calculated from the $B(x)$ by using Pagels-Stocker formula are shown in Fig. 2.

From these figures we see that the behaviors of $B(x)$ are divided into two pieces. One is *Yang-Mills type* to which flat cases with $\delta = 1, 2$ and $y^* = 0$ brane in warped case (w_0) belong, where the zero mode gauge boson dominates DSB. Another is gauged *NJL type* to which flat cases with $\delta = 3, 4, \ldots$ and $y^* = \pi R$ brane in warped case (w_π) belong, where the gauge boson KK modes dominate DSB. The dynamical masses are given by $B(\Lambda_{\mathrm{QCD}}^2)$, $f_\pi \sim \Lambda_{\mathrm{QCD}}$ ($\delta = 1, 2; w_0$), $\Lambda_{5\mathrm{D}}$ ($\delta = 3, 4$), $e^{-\pi kR}\Lambda_{5\mathrm{D}}$ (w_π). We observed almost no dependence on N_{KK} ($< 10^2$), that means the power low running acts as a suppression factor for DSB.

In summary, a chiral phase structure (nonperturbative dynamics) is nontrivial on the orbifold. For the orbifold QED, α_c decreases as $N_{\mathrm{KK}} \sim R\Lambda$ increases. For the orbifold QCD, we have two sorts of the result. One is the Yang-Mills type ($\delta = 1, 2$ and w_0) and the other is the gauged NJL type ($\delta = 3, 4, \ldots$ and w_π). We may have to be careful about such dynamically generated masses in various kinds of phenomenological models on the orbifold. As future works, a detailed analysis with a nonlocal gauge fixing (e.g., Appendix in Ref. [11]), with more precise one-loop running coupling (e.g., Ref. [12]) and/or with different regularizations (e.g., Ref. [13]) may be important. It is also interesting to consider a quasi-localized fermion ($|M_{\mathrm{kink}}| < \Lambda$), a supersymmetric case (e.g., 5D $\mathcal{N} = 1 \to$ 4D $\mathcal{N} = 2$, instanton, duality, etc.), and an application to a dynamical electroweak symmetry breaking (e.g., Ref. [14]).

This talk is based on Refs. [11, 15, 16].

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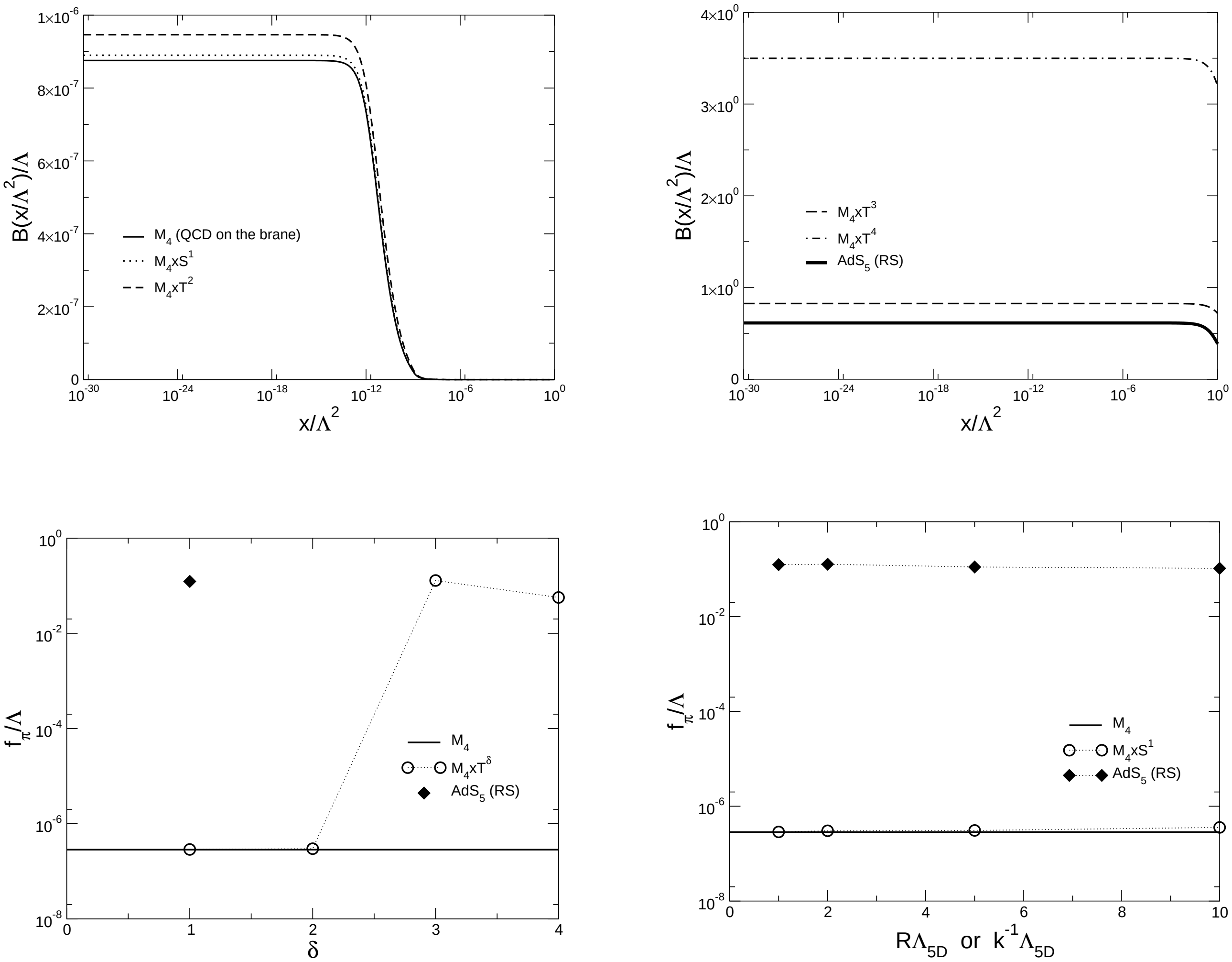

Figure 2: Dynamical mass B and f_π on a boundary of bulk QCD ($\Lambda_{\mathrm{QCD}}/\Lambda = 2 \times 10^{-5}$).

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